

Operator Theory
Advances and Applications
266

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Multivariate Prediction, de Branges Spaces, and Related Extension and Inverse Problems

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